MINISTRY OF EDUCATION AND TRAINING
HCMC UNIVERSITY OF TECHNOLOGY AND ENGINEERING

PHAN THANH NHAN

CHARACTERIZATION AND MODELLING VISCOELASTIC AND VISCOPLASTIC BEHAVIOR OF ASPHALT MIXTURES IN LOW STRAIN REGIME

Major: Mechanical engineer
Code: 9520101

SUMMARY OF DOCTORAL DISSERTATION

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Supervisor: Dr. Nguyen Huynh Tan Tai

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Reviewer 3:

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CHAPTER I: INTRODUCTION

1.1 Background

Rutting is one of the most popular distresses of asphalt pavement which caused by accumulation of irreversible deformation. Based on experimental observations, Di Bendetto et al. (2007) classified the behavior of asphalt concrete into four main types depending on temperature, magnitude of strain/ stress and time (cycle) loading includes: linear viscous elastic (LVE), nonlinear viscous elastic, irreversible deformation (rutting), and fatigue. In this study, PhD students will focus on investigating irreversible deformation of asphalt mixture. Since the deformation of asphalt mixture includes a part of reversible deformation and a part of irreversible one, research on reversible deformation or viscous elastic behavior is also important.

1.2 Literature review

1.2.1 Permanent deformation models of hot asphalt mixture pavement

Numerous models were proposed to predict the permanent deformation (rutting) of asphalt mixtures which can be typically categorized into analytical and constitutive models. Analytical models predict require a lot of financial and time costs to calibrate their parameters and can not simulate load history and hardening-relaxation behavior. The LVE properties of previous proposed constitutive models were determined in frequency domain so these properties must be converted to time domain before implanting them into calculating software. This process increases the calculation costs and decreases the accuracy of LVE properties of material. Moreover, algorithms to determine irreversible strain in those studies are not high accuracy. Therefore, the result of predicting rut depth of those models has a rather large difference with the actual measurement value.

1.2.2 Asphalt mixture laboratory characterization

Present laboratory testing to determine VP properties of hot asphalt mixture has some disadvantages such as the stress and strain fields are not homogenous (eg. Hamburg Wheel Tracking test), measured strains consist of a part of VE strain which is not reversible yet (eg. load permanent deformation test), and accuracy of algorithms to determine the VE strain are not good. Therefore, it is necessary to develop a testing method and corresponding algorithms for determining both LVE and VP properties of asphalt mixture have a complex history of loading time

1.2.3 Converting linear viscous elastic properties of material from frequency domain to time domain

Converted LVE properties in the frequency domain to the time domain is a nonlinear least-square problem. For easier to solve, it may be chosen the priori to convert the problem to the linear one or using an analytical approximation. That lead to errors of interconverting methods can reach to 5% as reported by Emri et al. (2005). Thus, development of an algorithm to convert LVE properties in the frequency domain to the time domain with high accuracy should be considered.

1.3 Research objective

The objective of this work includes: (i) development of an algorithm to convert LVE properties in the frequency domain to the time domain with a high accuracy and sum of exponential functions form; (ii) development of a generalized CR test and a back-calculation method for determining the permanent deformation of asphalt mixture in the time domain; (iii) investigating the VP properties of asphaltic material by using generalized CR tests; (iv) development of a viscoplastic model for predicting the VP behavior of asphalt materials.

1.4 Research scope: this study focuses on investigating the permanent strain of the asphalt concrete layers but ignores the deformation of the granular and subgrade

1.5 Research methodology: Three main research methods in this study are theoretical methods, experimental methods, and modeling methods.

1.6 Dissertation organization

This dissertation consists of six chapters, including: Chapter I gives a background and literature review for determining the objectives of this dissertation; approaching and methodological research are also introduced in this chapter. Chapter II presents a brief LVE, VP theory and synthesis the experimental methods, proposes an interconversion LVE properties in frequency domain to time domain with high accuracy, and generalizes CR test that can determine both pure VE, VP strain just in one test. In Chapter III, applying the algorithm of GCR test to validate the Time-Temperature superposition still held in VP domain. Chapter IV investigates the plastic yield point of asphalt mixtures by using slop creep-recovery test. Chapter V characterizes and models VE and VP behavior of asphaltic materials. Chapter VI summarizes the findings and guide for future research.

CHAPTER II: THEORATICAL BACKGROUND

2.1 Time-Frequency interconversion methods

In literature, plenty of time-frequency interconversion methods were proposed. Those methods are difficult to perform or unsatisfied with Kramer-Kronig relationships, or low accuracy. Therefore, in this subsection, the proposed method which fully obeys the physical causality principle and LVE theory and has a high accuracy is presented.

2.2.1 Algorithm for determining the RM and CC in the form of sums of exponential functions (SEFs) from dynamic modulus (DM) master curves

Input data: presmoothed (mathematical or mechanical) master curves of DM data:

- Define the range of frequency of interest (ω_i) in the logarithmic scale.

- Define the range of time of interest (t_i) in the logarithmic scale.

Step 1. Numerically calculate the phase angle (PA) $\varphi_E(\omega_i)$, then calculate the storage modulus $E'(\omega_i) = |E(i\omega_i)| \cos\varphi_E(\omega_i)$

Step 2. Decompose with high accuracy the storage modulus into SEFs $E'(\omega) = E'(\omega \to \infty) - \sum_{j=1}^{N_j} E'_j e^{b_j \omega}$

Step 3. Calculate the RM: $E(t) = E^*(\omega \to \infty) - \frac{2}{\pi} \sum_{i=1}^{N_j} E'_j \tan^{-1}\left[t/(-b_j) \right]$

By similar procedure with storage creep compliance, we have:

$$D(t) = D^*(\omega \to \infty) + \frac{2}{\pi} \sum_{i=1}^{N_j} D'_j \tan^{-1}\left[t/(-c_j) \right]$$

2.2.2 Application examples

Two application examples were performed to evaluate the efficiency of the proposed algorithm. For validation purposes, the obtained results will be compared with either exact solutions, analytical approximate solutions, or experimental results.

2.2.2.1 Master curves of dynamic modulus of HS model as input data

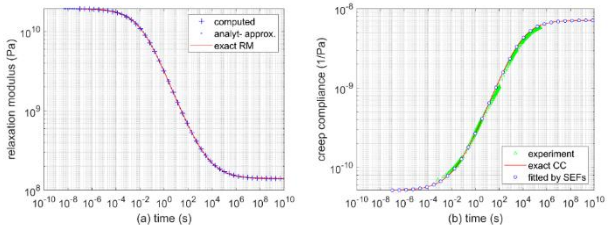

Figure 2.12. (a) Computed relaxation moduli from HS dynamic modulus and their approximations by SEFs; and (b) computed CCs from HS dynamic modulus and their approximations by SEFs.

The RM and CC were computed using the closed-form formulas in the Eq (2.65), the Eq (2.66) and are presented in Figure 2.12. The results show that the relative errors of the obtained RM and CC were lower than 3.46×10^{-4} at all computing points. In addition, the master curve of CCs of the mixture experimentally determined by Nguyen et al. (2021a) is also shown in the figure to confirm the good agreement between the computed and experimental CCs.

2.2.2.2 Master curves of dynamic modulus of GS model as input data

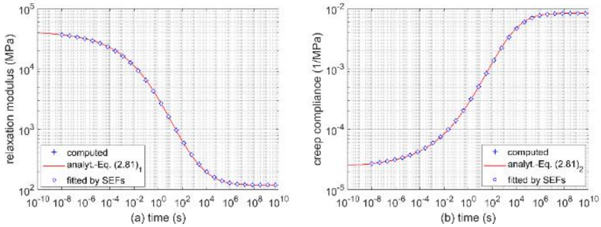

Figure 2.14. (a) Relaxation moduli computed from the GS master curve of DM and their approximations by SEFs; and (b) creep compliances computed from the GS master curve of DM and their approximations by SEFs.

The storage modulus and compliance were computed then approximated by the SEFs. The number of exponential functions used was 97 for the storage modulus and 74 for the storage compliance, which allows an extremely small relative error of lower than 8×10^{-8} at all estimating points. The RM and CC were then obtained by using the closed-form formulas in Eq (2.65 and 2.66) and are presented in Figure 2.14. The analytical approximate results are also presented in the figure to verify the agreement between the results obtained with the two different approaches. It was found that the relative difference between the results is rather small, lower than 2.7×10^{-2} at all estimating frequencies. Thus, the proposed algorithm allows determining reasonable values of LVE properties based only on the master curve of the DM.

2.2.3 Conclusion

A novel method for determining with high accuracy the RM and CC of asphaltic materials in the form of SEFs based only on the master curves of DM has been successfully developed. In the formulation, the PA was numerically calculated from the DM master curve on the basis of the exact Kramers-Kronig relationships. Owing to the special form of the SEFs used for approximating with high accuracy the storage modulus and compliance, the inverse Fourier transform was analytically performed to derive the closed-form formulas of the RM and CC. Thus, the proposed formulation can be considered to fully obey the LVE theory and physical causality principle, giving rise to an excellent level of accuracy of the obtained results. The main findings of this work can be drawn as follows: (i) The relative error of the PAs calculated was very small despite a rather coarse frequency step that was used, namely lower than 1.75×10^{-4} at all computing points; (ii) The maximal relative error of the approximation by SEFs using the modified Dombi algorithm was extremely small, namely lower than 8×10^{-8} when using both negative and positive coefficients, which can be considered neglected. (iii) The inverse Fourier transform was exactly performed to calculate the RM and CC thanks to the special form of the SEFs. (iv) The computed RM and CC were also approximated with high accuracy by SEFs using the modified Dombi algorithm. The maximal relative error was lower than 3.8×10^{-5} when using only positive coefficients.

The level of the overall accuracy of all computed LVE properties was dominated by that of the PA. Therefore, more accurate results can be obtained by just improving the accuracy of the numerical integration in the PA calculation step. Although the proposed algorithm for determining the RM and CC in the forms of SEFs requires more calculation steps, the obtained results are much more accurate, which will contribute to better analysis and design of asphaltic pavement, and therefore a longer service life.

2.3 Laboratory testing for determining linear viscoelastic properties.

For LVE properties of asphaltic material, they can be investigated in both the time and the frequency domain. In the time domain, laboratory testing methods include relaxation test, creep test, creep – recovery (CR) test. In this dissertation, the algorithm which proposed by Nguyen et al. (2021) is chosen to process data of CR tests because that algorithm can determine the CCs of asphaltic material at intermediate and high temperatures with high accuracy. The dynamic modulus test is the widely used test to determine LVE properties in the frequency domain because of its stabilization and simplicity.

2.4 Laboratory testing for determining permanent deformation

2.4.1 Algorithm for determining the pure VP strain based on the GCR test data

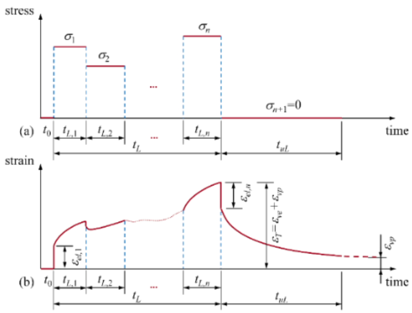

Figure 2.22. Description of the GCR test, (a) applied stress history, (b) response strain (Tran et al, 2022).

Input data: Time histories of stress, measured strain, number of the loading period *n*

- Calculate the experimental recovery compliance using Eq (2.105)

$$\frac{\varepsilon_T(t_n)-\varepsilon_T(t)}{\sigma_n}=\sum_{i=0}^{n-1}D(t_n-t_i)\frac{\Delta\sigma_i}{\sigma_n}-\sum_{i=0}^{n-1}D(t-t_i)\frac{\Delta\sigma_i}{\sigma_n}+D(t-t_n),\ \text{for } t\geq t_n \quad (2.105)$$

- Calculate the discrete stress increment $\Delta\sigma_i, i\in\{0,n-1\}$

Step 1. Back-calculating the CC:

- Choose a set of initial values for analytical approximate CC (AACC) function derived from HS model.

- Use curve fitting techniques to best fit Eq (2.105). The AACC function with the best-fit parameters is the solution to Eq (2.105) using the least square means.

Step 2. Predicting the pure VE strain:

Calculate the VE strain using the best-fit parameters of the AACC function determined in Step 1:

$$\varepsilon_{ve}(t)=\sum_{i=0}^{m,t_m<t\leq t_{m+1}}D(t-t_i)\Delta\sigma_i,\ \text{for } t_0<t\leq t_n \quad (2.99)$$

$$\varepsilon_{ve}(t) = \sum_{i=0}^{n-1} D(t - t_i)\Delta\sigma_i - D(t - t_n).\sigma_n, \text{ for } t > t_n \tag{2.100}$$

Step 3. Computing the pure VPstrain:

Calculate the difference between the measured strain and the predicted VE strain: $\varepsilon_{vp}(t) = \varepsilon_T(t) - \varepsilon_{ve}(t)$

2.4.2 Validation method

By conducted single-stress level creep-recovery (SSCR) test, the results showed that values of VP strains calculated with present algorithms are also more accurate than those computed with Abu Al-Rub et al. (2009)'s algorithm (see Fig 2.29).

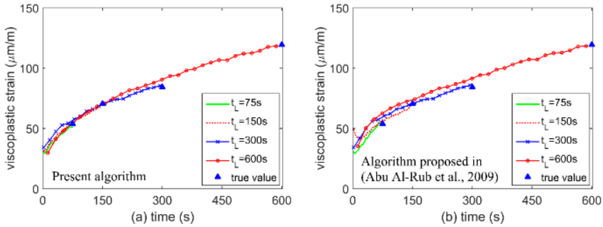

Figure 2.29. Pure VP strains obtained from all SSCR test replicates at 50°C, (a) using the AACC function, (b) using the CC function in the form of a Prony series.

Moreover, ascending and descending order of a loading period of five equal time steps were investigated for verification of the performance of GCR test in handling with a complex loading history. After eliminating the VE strains from the measured strains, the VP strains of all increasing MSCR test replicates were obtained and presented in Figure 2.37. Because the initial PD behavior of the material in all test replicates is different, it cannot be basically compared the obtained VP strain to each other. However, it is still considered meaningful for comparison if the resistance of PD does not change so much. In the case of this study, the RDs do not exceed 10.91% between any curve of VP strain and the average curve, therefore, it can be considered that the calculated VP strains repeatedly. Moreover, the relative difference between the obtained VP strains and the ‘true values’ is lower than 4.78% which demonstrates the good reliability of VP strain values calculated based on the proposed method even with a more complicated loading history.

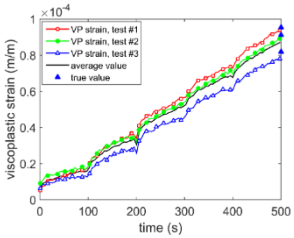

Figure 2.37. Evolution of VP strains at 50°C in sample #3 during all increasing MSCR test replicates

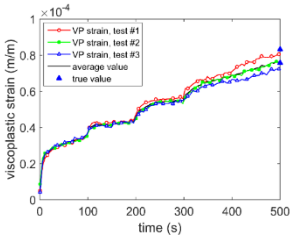

Figure 2.41. Evolution of VP strains at 50°C in sample #3 during all decreasing MSCR test replicates.

Figure 2.41 shows the VP strains determined from the data of all decreasing MSCR test replicates. The DR between any curve of VP strain and the average curve is lower than 4.35%. In addition, the computed VP strains differ less than 4.41% from the ‘true values’. Therefore, the proposed method is capable of determining reliable VP strains even with a complicated loading condition like that in the decreasing MSCR test.

2.4.3 Conclusion

A generalised version of the CR test procedure has been proposed to examine the PD of asphalt mixtures exerted by any complex loading history in the time domain. The results of the GCR test were successfully used to characterise both the VE and PD behaviour of the material at 50°C without conducting DM tests. Owing to the present back-calculation algorithm, the CCs were determined with a good level of accuracy and repeatability, based on which the viscoplastic strains could be accurately determined even with short recovery periods. The main findings can be drawn as follows: (i) The AACC function derived from the HS model outperforms the Prony series CC function when used to fit the recovery compliance data. The CCs obtained with the latter function deviated far from the exact CCs, especially at low loading times, while those obtained with the creep function derived from the HS model showed a good agreement with the exact CCs; (ii) The duration of the recovery period has a significant impact on the directly measured VP strains at 50°C; however, it does not affect much the back-calculated VP. In fact, a resting time equal to 2–3 times the loading time could lead to an accuracy level of roughly 3% for the back-calculated VP strains. Therefore, a ratio of resting time to loading time ranging from 2 to 5 was sufficient for the back-calculation algorithm to give reliable results for tests at 50°C. Conversely, a much longer resting period was needed for the direct measurement to achieve good results at the same testing temperature, at least 7 times the loading time; (iii) At a high temperature (50°C), permanent strains account for an important part of the total strains, which suggests that a mixture with higher stiffness may have a higher resistance to rutting. (iv) When the resting time equals 5 times the loading time, the relative errors of the VP strain are lower than 4.78% while any curve of VP strain differs < 10.91% from the average curve, which proves that the back-calculation algorithm is reliable at the high testing temperature (50°C); (v) The loading history does affect the development of VP strain. With the same number of loading intervals, duration of loading period, and maximal level of the applied stress, the VP strain obtained in the increasing MSCR test was larger than that obtained in the decreasing MSCR test, approximately 14.16% higher in the conditions studied.

According to the obtained results, the proposed test procedure allows determining with good accuracy and repeatability the pure permanent deformation of asphalt mixtures. Therefore, the results can be used to calibrate predictive models as well as to evaluate the rutting potential of asphalt mixtures.

CHAPTER III: VERIFICATION OF TIME-TEMPERATURE SUPERPOSITION PRINCIPLE FOR VISCOPLASTIC STRAINS OF ASPHALT MIXTURES

3.1 Introduction

The objective of this section is to adequately verify the applicability of Time-Temperature Superposition Principle (TTSP) in the VP domain within the low-strain regime. The reason for limiting the strain amplitude is to ensure that the LVE theory is valid and the LVE properties in both time and frequency domains can be accurately determined using fundamental procedures, based on which the reliability of the obtained results in the VP domain can be assessed. The specific objectives of this work are defined as follows: (i) investigating the pure VP strains in the low-strain regime of a variety of asphalt mixtures caused by several stress levels and temperatures using CR tests; (ii) developing new theoretical formulations to quantitatively estimate the influences of various factors causing biases in the obtained results; (iii) examining the extension of existing LVE shifting techniques into the total and VP strains and developing a new shifting method to reliably determine the shift factors; (iv) analysing the properties of the obtained master curves and their corresponding shift factors to theoretically demonstrate the applicability of TTSP in the VP domain; (v) applying a simple VP model for predicting their irreversible strains of all studied mixtures to demonstrate the efficiency of the master curve of VP strains and the companion VP shift factors in characterising and predicting the PD behaviour of asphalt mixtures, based on which the TTSP in the VP domain can be experimentally validated.

3.2 Material and method

In this section, the application of TTSP in LVE and VP domain of three asphalt mixtures including a conventional mixture (AC), a polymer-modified bitumen (PMB) asphalt mixture, and a highly modified asphalt mixture (HiMA) were examined. The VP strains of SSCR tests can be calculated using the algorithm presented in subsection 2.4.1. Those VP strains then are shifted to construct master curves by using a new proposed shifting method which algorithm presented as below

Algorithm for determining the shift factor

Step 1: Determine the shift factor $\left(a_T^{n+1,n}\right)$ applied for $(n+1)^{th}$ isotherm according to n^{th} isotherm

- Fitting each data set by a (quadratic) polynomial, noted $f_n\left[\log(t)\right]$; $f_{n+1}\left[\log(t)\right]$

- Given an initial value of $a_T^{n,n+1}$, calculating the value of $f_{n+1}\left[\log\left(t / a_T^{n+1,n}\right)\right]$

- Determining $a_T^{n+1,n}$ by minimising:

$$\Delta f^2 = \sum_{j=j_1}^{j_2}\left\{f_n\left(t_j\right) - f_{n+1}\left[\log\left(t_j\right) - \log\left(a_T^{n+1,n}\right)\right]\right\}^2 ,$$

wherein $f_n\left[\log\left(t_{j_1}\right)\right]$ and $f_n\left[\log\left(t_{j_2}\right)\right]$ define the potential overlap region of the two isotherms

Step 2: Determining the shift factor applied for an arbitrary isotherm according to the isotherm at T_{ref}

- Let the k^{th} isotherm be the reference isotherm; the shift factor applied for an arbitrary n^{th} isotherm according to the isotherm at T_{ref} :

$$a_T^{n,k} = \prod_{i=k}^{n} a_T^{i,i+1} \text{ if } n \geq k; \; a_T^{n,k} = \prod_{i=n}^{k} a_T^{i,i+1} \text{ if } n < k.$$

3.3 Experimental result and analysis

In this section, the results of twenty-one SSCR test for each mixture were used to verify the TTSP in viscoplastic domain and to check replicate of the results. Each SSCR test consisted of 150s loading time and 3000s resting one. Each sample was applied one stress level (23.81, 35.71, or 53.57kPa) at four different temperatures to eliminate the effects of the heterogeneity of asphalt mixture on obtained results. Testing temperatures are 17.5, 25, 32.5, and 40°C for AC mixture; 27.5, 35, 42.5, and 50°C for PMB mixture; and 40, 47.5, 55, and 62.5°C for HiMA mixture.

3.3.1 Master curve of VP strains

Figures 3.18 - 3.20 present shifted curves of VP strains of investigated mixtures and their corresponding shift factors noted as $a_{T,VP}$. Figures 3.18 – 3.20 (a) showed that form of those curves are relatively smooth and unique. Moreover, with the monotonous shapes, those curves satisfy a unique relationship between time and VP strain. Therefore, it can be considered that the shifted curves of VP strains are their master curves at the reference temperature. Furthermore, Figures 3.18 – 3.20 (b) show that all shift factors ensure an injective relationship between temperature and $a_{T,VP}$ because their functions are also monotonous functions. It means that TTSP can be applied on VP strains of study mixtures.

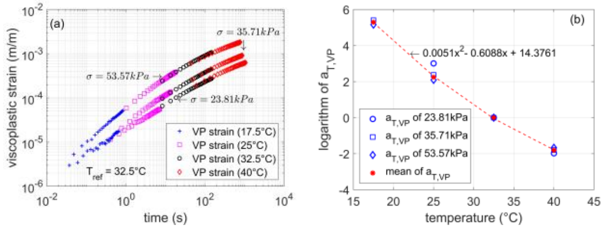

Figure 3.18. Master curves of all VP strains of AC mixtures (a) and their corresponding shift factors (b).

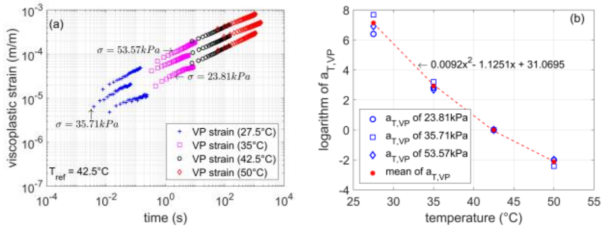

Figure 3.19. Master curves of all VP strains of PMB mixtures (a) and their corresponding shift factors (b).

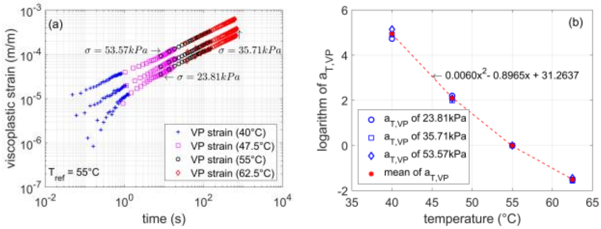

Figure 3.20. Master curves of all VP strains of HiMA mixtures (a) and their corresponding shift factors (b).

3.3.2 Comparison of total, VE, and VP shift factors

Figure 3.21 plotted the values of $a_{T,T}$ (shift factor of measure strains) , a_T (shift factor of VE strains), and $a_{T,VP}$ together for comparing them. Effect of cumulative VP strain and uninstantaneous applied load in the loading period of CR test were also investigated to determine the reliable intervals of the shift factors.

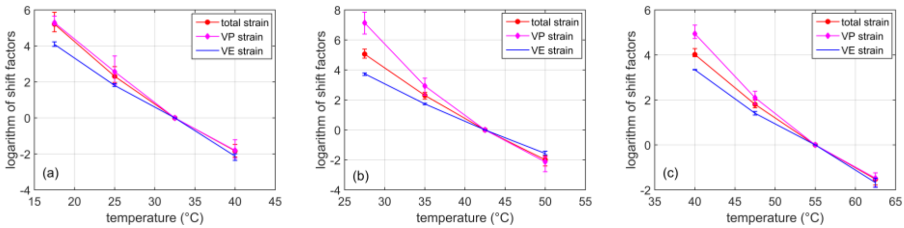

Figure 3.21. Shift factors of total, VE, and VP strains obtained with (a) AC, (b) PMB, and (c) HiMA samples.

It can be observed in Figure 3.21 that value of $a_{T,T}$ and $a_{T,VP}$ seem to be similar to value of a_T at high temperatures. This explains why many authors suggest extending the validity of VE strain shift factor into the VP domain. Nevertheless, it is easy observed that values of total strain or VP strain shift factors are not similar to values of a_T in the full range of study temperatures. In case eliminating all of factors causing biases to values of shift factor, between $a_{T,T}$ and a_T, and between $a_{T,VP}$ and a_T still exist clearly recognisable gaps. It means that the $a_{T,T}$ and $a_{T,VP}$ are independent on a_T as suggested by Cao & Kim (2022).

3.4 Conclusion

In this section, VE and VP behaviour of three asphalt mixtures including AC, PMB and HiMA mixtures were successfully characterised using SSCR tests and the companion back-calculation algorithm. Owing to the proposed shifting method, the VE and VP shift factors were separately identified and demonstrated to be obviously different. With the obtained shift factors, the master curves of CCs and VP strains were constructed. The results showed that the CCs obtained with SSCR test agreed well with those interconverted from dynamic modulus data, which confirms the reliability of the obtained VP strains. All master curves of VP strain were relatively smooth and monotonous, ensuring an injective relationship between time and VP strain. In addition, all VP shift factors were independent of the stress levels applied and injective with the temperatures. Therefore, TTSP still holds in VP domain, at least with the examined asphalt mixtures.

Owing to the results of this work, the VP behaviour of asphalt mixtures can be investigated over a wide range of loading time by conducting a series of SSCR tests with reasonable testing times instead of performing extremely long-time SSCR tests. Once the master curves of VP strain are constructed, the resistance to PD of asphalt mixtures will be assessed with high sensitivity. In fact, AC mixture was demonstrated to be the most vulnerable to PD, compared to PMB mixture and HiMA mixture. Furthermore, the master curves of VP strain can be used to calibrate rutting prediction models, which will be carried out in future works.

CHAPTER IV: CHARACTERIZATION OF ASPHALT MIXTURE VISCOPLASTIC YIELDING

4.1 Introduction

The characteristic of the yield surface applied to asphalt concrete is crucial to properly predict the accumulation of permanent deformation strain. However, the proposed yield stress identification methods are controversial and do not convince readers. Therefore, the objective of this section is to propose a method capable of identifying the viscoplastic yield stress—if it exists—within the low-strain regime.

4.2 Theoretical framework

In this section, the slope creep-recovery (SCR) test was used to characterize the viscoplastic yield stress of an asphalt concrete (AC) mixture. A ramp function of the stress was applied during a certain duration before being removed, for the strain to recover a part of its reversible part. The GCR algorithm proposed in subsection 2.4.1 was used to post-process SCR test data with the loading period of the stress history be divided by *n* subintervals. The viscoplastic yield stress is determined based on obsevering the obtained VP strains curve as the following equation.

$$\begin{cases} \dot{\varepsilon}_{vp}(t) = 0, \text{ for } \sigma \le \sigma_y \\ \dot{\varepsilon}_{vp}(t) > 0, \text{ for } \sigma > \sigma_y \end{cases} \tag{4.2}$$

4.3 Experimental program and analysis of results

In this section, AC mixture was chosen for investigating the viscoplastic yielding because it is widely used for the road system in Vietnam

4.3.1 Convergence of the GCR algorithm

Before moving on to the assessment of viscoplastic yield stress, we performed a quick analysis on the convergence of the GCR algorithm to determine a balanced level of discretization in terms of accuracy and computational cost. As can be observed in Figure 4.4(a), the relative errors exhibit a decreasing trend with an increasing number of subintervals. In terms of computing time, a finer discretization clearly requires a longer computation time to complete the calculation, as shown in Figure 4.4(b). According to the results obtained, a discretization using 320 subintervals appeared sufficient to ensure both numerical accuracy and computational efficiency.

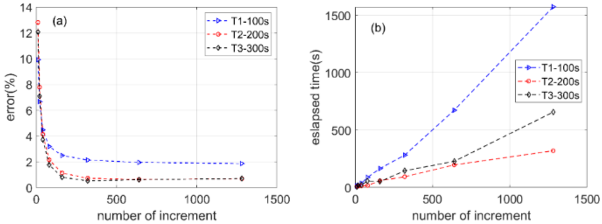

Figure 4.4. Effect of the discretization fineness on (a) the accuracy of the results obtained with the GCR algorithm, and (b) elapsed time for post-processing.

4.3.2 Verification of the non-existence of viscoplastic yield point

Regarding the evolution of VP strains, it can be observed in Figure 4.5(b) that all VP strain curves continuously increased from the onset of stress application. This may suggest that the material does not exhibit viscoplastic yield points at 30 °C.

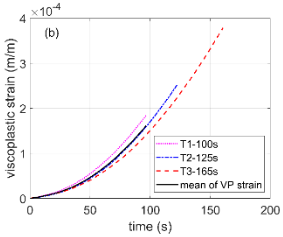

Figure 4.5. (b) Evolution of VP strains of all tests performed on sample #1.

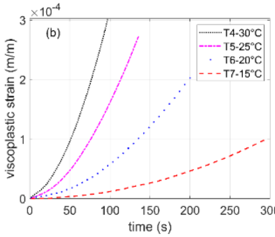

Figure 4.6. (b) Evolution of VP strains of all tests performed on sample #2.

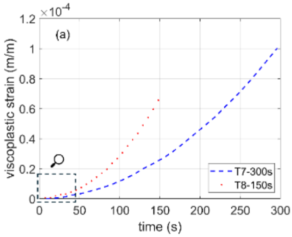

Figure 4.7. (a) Evolution of VP strains at 15°C

Figure 4.6(b) shows that the obtained VP strains at at different temperatures tend to increase continuously with the loading time due to increases in the stress magnitude applied, except for the VP strain obtained at 15°C with a loading rate of 3N/s. To verify if there exists a viscoplastic yield point for the studied mixture at 15 °C, an additional test (T8) was conducted at the same temperature but with a higher loading rate of 6 N/s and the obtained VP strains showed an increasing trend despite relatively large oscillations. This proves that the viscoplastic yield point of the investigated mixture at 15 °C—if it exists—is extremely small and can therefore be considered negligible.

4.4 Conclusion

The viscoplastic yield point of a conventional asphalt mixture was successfully characterized by using slope creep-recovery tests in the low-strain regime. It is efficient to apply the generalized creep-recovery algorithm for separating viscoelastic and viscoplastic behavior of the specimen from slope creep-recovery test data. Based on the identified VP strain curves, the material did not show any observable separation point between reversible and irreversible regions within the testing temperatures, ranging from 15°C to 30°C. This finding is in line with the assumption made by many other authors, yet has not been properly verified. The results of this study will support the development of permanent deformation behavior models based on non-yielding viscoplastic theory.

CHAPTER V: MODELLING BEHAVIOR OF ASPHALT MIXTURE UNDER UNIAXIAL COMPRESSIVE STRESS

5.1 Introduction

As reported in subsection 1.2.1, previous proposed models have some disadvantages such as: consuming plenty of financial and time costs, cannot consider effect of loading and resting duration, applied stress, or temperature, do not consider the hardening – relaxation effect, assumed that asphalt mixture has the plastic yield point. Therefore, in this chapter, rules of permanent hardening and phenomenon of material recover its ability to develop VP strains in resting period are investigated before a new model is proposed for effective predicting the evolution of VP strains under complex applied stress paths.

5.2 Proposed viscoplastic model

By adopting the Perzyna's VP flow rule, the strain rate at any time is written as:

$$\dot{\varepsilon}_{VP}(t) = h(P)\cdot\xi(s)\cdot sign(f(\sigma))\cdot|f(\sigma)|^{m} \tag{5.3}$$

where, *h(P)* represents the "permanent mechanical hardening" effect, $\xi(s)$ is time-related relaxation parameter representing the "actual capability" of the material in producing viscoplastic strain rate of the material, and $f(\sigma)$ is the loading function. It has been observed by many authors and confirmed via experiments in this work that asphalt mixture does not show obvious viscoplastic strength yield as will be demonstrated in chapter IV; thus, the loading function can be written as $f(\sigma)^{m} = \sigma/\sigma_L$, for $\sigma \le \sigma_L$ and $f(\sigma)^{m} = (\sigma/\sigma_L)^{m}$, for $\sigma > \sigma$ where *m* >1 is a parameter used to represent the nonlinearity effect of the stress on the evolution of VP strain. It should be noted that *m* can also be a stress-dependent parameter. For instance, the following model was used to predict the dependency of power coefficient $m = 1 + n$, for $\sigma \le \sigma_L$ and $m = 2 + n - 1/(1 + m_1\sigma^{m_2})$, for $\sigma > \sigma_L$ where *n* is the power coefficient in equation of the fluidity function . The evolution of variable *s* is characterized by equation $s = \int_0^t D_s(t-\tau)\dot{\sigma}d\tau$ .The function $D_s(t)$ was chosen as power function, taking the form: $D_s(t) = A_1t^{q_1} + A_2t^{q_2}$. The permanent hardening law and the fluidity function can be predicted by equation $h(P) = h_1 + h_2/(1 + h_3\cdot e^{h_4P})$ and equation $\xi(s) = \xi_1 + \xi_2/(1 + \xi_3\cdot s^{n})$.

5.3 Model calibration

This section presents a systematic analysis procedure to calibrate VE, and VP model. Raw obtained data of tests were analyzed based on algorithms proposed in subsection 2.4.1. Back calculated CCs were used to calibrate parameters of VE model (HS model) while parameters of VP model were determined based on the computed VP strain.

5.3.1 Identification of HS model

Parameters of HS model were calibrated by fitting function of HS model to the average CC which is mean of back-calculated CCs. Value of the model parameters are presented in Table 5.4.

Table 5.4:Parameters of HS model

Param	E_0(MPa)	E_∞(MPa)	h	k	δ	τ
Value	109.929	38319.305	0.525	0.435	912.613	0.185

5.3.2 Identification of the parameters of VP model from two cycle CR test

Table 5.5 presents parameters of permanent hardening function *h(P)* were identified by fitting with experimental result. Function's parameters of time-related relaxation behavior and nonlinear parameters of VP strains are identified by minimizing the total errors simultaneous fit to calculated VP strains of test #1÷8 and #15÷18 conducted on sample #1. Results are displayed in Table (5.7 – 5.8).

Table 5.5: Parameters of *h(P)* obtained from tests on sample # 1

Param	h_1	h_2	h_3	h_4
Unnormalized	0.222	1620.758	1786.121	1462.376
Normalized	0.196	1435.441	1786.121	1462.376

Table 5.7: Parameters of time-related relaxation function

Param	ξ_1	ξ_2	ξ_3	A_1	q_1	A_2	q_2
Value	0	$9.684\text{x}10^{-6}$	7.297	$1.039\text{x}10^{-5}$	0.613	$1.128\text{x}10^{-5}$	0.9

Table 5.8: Parameters of *m* and nonlinear of VP strains

Param	m_1	m_2	n
Value	1.127	$1.144\text{x}10^{-3}$	0.5

5.4 Model validation

Once the LVE and VP model parameters are determined from data test conducted on sample #1, the predictions of model will be compared with independent laboratory test results performed on sample #2 for evaluation of the efficacy of the model. Predicting results of two cycle creep – recovery (CR-TC), various applied stress and loading time creep – recovery (CR-VLT) and slope creep – recovery (SCR) tests were presented in Fig 5.9 - 5.11

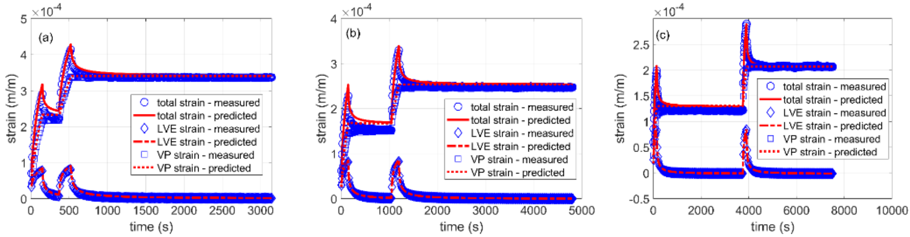

Figure 5.10. The comparison between model predictions and experimental measurements for CR-TC test. (a) test CR-TC-225s, (b) test CR-TC-900s, (c) test CR-TC-1h.

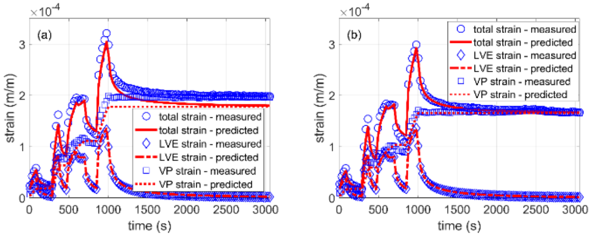

Figure 5.11. The comparison between model predictions and experimental measurements for CR-VLT tests. (a) test CR-VLT-R1, (b) test CR-VLT-R2.

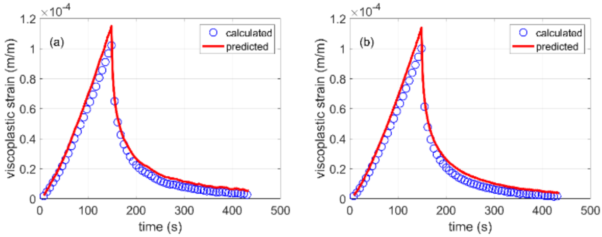

Figure 5.12. The comparison between model predictions and experimental measurements for SCR tests. (a) test SCR- R1, (b) test SCR- R2.

5.5 Conclusions

This study successfully proposes a model to predict the behavior of the considered asphalt mixture (AC) at 32.5 °C under uniaxial compressive stress, with the following contributions: (i) identifying the material's permanent hardening law based on experimental data; (ii) investigating the time-related relaxation phenomenon for VP strain; and (iii) developing a novel viscoplasticity model for asphalt materials in compression that incorporates permanent hardening and time-related relaxation functions.

Two cycle CR, various applied stress and loading time CR, and slope CR tests were applied to validate the model at 32.5°C shows that the proposed model has ability to predict the behavior of the studied asphalt mixture under complex loading paths. Like other predicted models, the applicability of this model is only valid within the investigating ranges of applied stress, loading and unloading period, and temperature wherein model parameters were identified. Although only one mixture was chosen in this chapter, the process was performed in a general manner and therefore the extension of the proposed method to other mixtures and temperatures could be performed in a similar manner, which will be demonstrated in future works.

CHAPTER 6: CONCLUSIONS AND RECOMMENDATIONS

6.1 Main conclusions

In this research, the behaviour of VE and VP of asphalt mixture were successfully characterised by using generalized creep recovery test and well predicted by a new proposed model. The main findings of this work can be drawn as follows:

First, it has successfully developed a novel method for interconverting the relaxation modulus and creep compliance of asphaltic materials in the form of SEFs based only on the master curves of dynamic modulus. Proposed method not only has an excellent level of accuracy but also fully obey the LVE theory and physical causality principle. The significant of that method is validation obtained VP strains in this study.

Second, the creep-recovery test procedure was generalized for determining both the viscoelastic and permanent deformation behavior without conducting dynamic modulus tests. It is worth noting that the proposed procedure can be applied to identify the permanent deformation of asphalt mixtures exerted by any complex loading history in the time domain. It is the foundation for investigating the viscoplastic properties of asphalt mixture.

Third, applying the single stress creep-recovery test with three stress levels and four temperatures on three asphalt mixture, obtained VP strains were shifted by a new shifting method modified of HonerKamp and Weese (1993) proposal. Results show that these are relatively smooth and monotonous for all master curves of VP strain to ensure an injective relationship between time and VP strain. Moreover, it is demonstrated that the stress levels applied do not affect on the VP shift factors and they are injective with the temperatures. Therefore, time-temperature superposition principle was verified still holds in VP domain, at least with the examined asphalt mixtures.

Fourth, slope creep–recovery tests conducted in the low-strain regime, combined with the GCR algorithm, enabled successful characterization of the viscoplastic yield point of the studied conventional asphalt mixture. Analysis of the resulting VP strain curves showed no clear distinction between reversible and irreversible regions across the testing temperature range (15–30 °C). This observation supports a common assumption in the literature that has not been thoroughly validated. These findings contribute to advancing VP models based on non-yielding viscoplastic theory.

Finally, a new viscoplastic model was proposed and coupled with Huet-Sayegh model to predict the behavior of an asphalt mixture under uniaxial compression state. The model incorporates permanent hardening law and time – related relaxation properties; thus, it is demonstrated that the model can predict the behavior of the investigated asphalt mixture at 32.5°C caused by complex loading histories in the low strain regime.

The results of this work rely on the following assumptions: (i) the VE behavior of materials remains within LVE domain, which restricts the application of large stress/strain magnitude; (ii) the total (measured) strains are additively decomposed into reversible and irreversible components. For the first assumption, nonlinear VE theory could be considered in the formulation to extend the applicability of the proposed method into large strain domain. Although the validity of the additive decomposition of strain was verified in the small strain regime, its extension into large strain domain should be verified in future works. Furthermore, the multiplicative decomposition of reversible and irreversible deformation gradient tensor, which is commonly used in large deformation for metallic materials, should be investigated.

6.2 Recommendations

Based on the results of this research, there are a few recommendations for future study to expand the applicability of model as follows:

In this study, the procedure of generalized creep-recovery test is limited to low strain regime so that the Boltzmann principle still validates for LVE behaviour of material. Hence, a suitable procedure should be developed for nonlinear viscoelastic behaviour of asphalt mixture.

The stress states of all tests in this research are uniaxial while the stress states of asphalt concrete layers are multi-axial. Thus, it is required to consider the effects of confining pressure in future works for expanding predicting ability of the proposed model.

The proposed model was calibrated and validated only with tests conducted on AC mixture. Consideration of changing mixture is necessary to evaluate the sensitivity of the model parameters.

LIST OF PUBLICATIONS

1. Phan, T.N.; Nguyen, D.L.; Vu, T.V.; Tran, V.T.; Nguyen, H.T.T; and Nguyen, M.L. Determining with High Accuracy the Relaxation Modulus and Creep Compliance of Asphaltic Materials in the Form of Sums of Exponential Functions from Mathematical Master Curves of Dynamic Modulus. Journal of Materials in Civil Engineering 34(9): 04022219. DOI: 10.1061/(ASCE)MT.1943-5533.0004366, 2022.
2. Phan, T.N.& Nguyen, H.T.T. Comparison the accuracy of approximate methods by sums of exponential using real-valued coefficients. Proceedings of 11th national mechanics conference. 513-521, 2022.
3. Tran, V.T., Phan, T.N., Tran, V.T., Do, T.T., Nguyen, H.T.T; and Nguyen, M.L. Development of a generalised creep-recovery test and a back-calculation method for determining the permanent deformation of asphalt mixtures in the time domain. Road Materials and Pavement Design. DOI: 10.1080/14680629.2023.2180291. 2023.
4. Nguyen, N.H., Phan, T.N., Do, T.T., and Nguyen, H.T.T. Evaluating the accuracy of linear viscoelastic properties in the time domain analytically approximated from complex modulus data . Proceedings of national mechanics conference for 45th anniversary of The Institute of Mechanics. 103-110, 2022.
5. Phan, T.N., Nguyen, N.H., Do, T.T., Nguyen, H.T.T; and Nguyen, M.L. Verification of time–temperature superposition principle for viscoplastic strains of asphalt mixtures obtained from creep-recovery tests in the low-strain regime. Road Materials and Pavement Design. DOI: 10.1080/14680629.2024.2390156. 2024.
6. Phan, T.N.& Nguyen, H.T.T. Characterizing the creep compliance of a conventional asphalt mixture using creep-recovery test. Proceedings of the 4th Annual International Conference on Material, Machines, and Methods for Sustainable Development (MMMS2024). 141-148, 2025.